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Modeling and Visualization of Complex Shaped Surfaces Using Interpolation Curves

E.V. Konopatskiy[1,A], A.A. Bezdytniy[2,B]

[A] Nizhegorodsky State Architectural and Civil Engineering University, Nizhny Novgorod, Russia
[B] Melitopol State University

[1] ORCID: 0000-0003-4798-7458, e.v.konopatskiy@mail.ru
[2] ORCID: 0000-0003-0528-9731, bezdytniy@gmail.com

Abstract

This article presents an approach to modeling and visualizing surfaces of complex shapes using interpolation curves with predetermined geometric properties. A modified Bezier curve of n-order was used as interpolation curves. Modification of a Bezier arc into an interpolation curve is possible both with and without preserving tangents. When preserving tangents, the Bezier arc retains its properties as a contour arc and acquires the ability to pass through preset points. The considered modification is possible in several variations: universal, based on the uniform distribution of the parameter during the modification process, and adaptive, when the parameter values are adapted to the initial data. The use of interpolation curves makes it possible to implement a special case of the moving simplex method, an analogue of which in geometric modeling and computer-aided design systems is the section operation (or lofting). The difference is that a continuous curve is used as a generating surface instead of a piecewise one. To ensure the functionality of such a connection, we give examples of models of the surface of an onion dome and a vase using various guides. An analysis of the obtained results was carried out. The introduction of research results into CAD/CAM will significantly expand their tools in terms of shape formation and visualization of surfaces and bodies that have predetermined geometric requirements.

Keywords: geometric modeling, surface, interpolation curves, moving simplex method, shaping operations, section operation, lofting.

1. Introduction

In modern systems of geometric modeling [1-3] and scientific visualization [4-8], the definition of surfaces with complex shapes plays an important role. Traditionally, both domestic and foreign geometric modeling systems distinguish 4 main form-building operations: extrusion, rotation, kinematic operation and sectional operation (lofting). The first 3 operations are usually classified as kinematic with a different trajectory of the guide line, which can be rectilinear, circular or arbitrary. A distinctive feature of the section operation is that the guide lines - sections that determine the trajectory of the generative line - can be different both in shape and position in space. The operation of shaping by sections is widely used in modeling structures in aircraft and mechanical engineering, and the lofting method often appears in domestic and foreign scientific works [9,10]. However, all of the above operations are kinematic and can be generalized by the moving simplex method [11, 12], the essence of which is the use of local simplexes to define geometric objects. Local simplices, moving in three-dimensional space, fill it with affine-equal lines, forming a continuous surface. The method has a generalization to multidimensional space and was effectively used for geometric modeling of multifactor processes and phenomena in the form of hypersurfaces of a multidimensional affine space [13]. At the same time, special interpolation curves were developed based on Bernstein polynomials, providing the shaping of surfaces and hypersurfaces of multidimensional space using the moving simplex method in point calculus. As a result of further research, these curves were improved [14] for more effective adaptation to the original data and the elimination of unplanned oscillations. In this case, the values of the current parameter and the distances between the interpolation nodes are actually matched, which provides adaptation of the interpolation curve to the original data on an irregular network of points. This approach in some cases can be an effective alternative to piecewise spline interpolation, which has found wide application in computer-aided design [15, 16], computer graphics [17] and visualization systems [18, 19].

2. Definition of interpolation curves with predetermined geometric properties

The definition of interpolation curves in point calculus was initially implemented using the example of Bézier curves of the n-order, which are analytically determined by Bernstein polynomials. The idea was to replace the control points of Bezier curves with interpolation nodal points by composing and solving a system of linear algebraic equations using the Cramer method. In this case, a uniform distribution of the parameter was used, providing the necessary replacement of points. In the case of a uniform distribution of interpolation nodes on a regular network of points, this approach gave a significant increase in productivity, since it made it possible to reduce most of the resulting parametric equations to linear ones. In addition, the linear relationship between coordinates (process factors) and parameters made it possible to easily implement the replacement of variables. At the same time, the starting points (interpolation nodes) are not always located equally. This leads to the need to use an uneven distribution of the parameter to determine adaptive interpolation curves [14].

Let's consider a method for determining interpolation curves based on Bezier curves. Based on this, the point equation of the Bezier curve arc of the n-order will have the following form:

$$M=\sum_{i=0}^{n} A_{i+1}\frac{n!}{i!\left(n-i\right)!}\bar{u}^{n-i}u^{i}, \tag{1}$$

where M – the current point of the Bezier curve, which with its movement fills the space, forming a Bezier curve line;

A_{i+1} – Bezier curve control points;

u – current parameter, which varies from 0 to 1;

$\bar{u}=1-u$.

To replace control points with interpolation nodes, it is necessary to determine the value of the u-parameter corresponding to each of the interpolation nodes. Let us accept these values based on the uniform distribution of the current parameter values $u=\frac{j}{n}$. As a result, we get:

$$M=\sum_{i=0}^{n} A_{i+1}\frac{n!}{i!\left(n-i\right)!}\left(\frac{n-j}{n}\right)^{n-i}\left(\frac{j}{n}\right)^{i},$$

In accordance with the methodology described in [10], points A_{i+1} are replaced by points M_{i+1} in the point equation (1) by composing and solving a system of linear algebraic equations. In this case, the condition is accepted that the curve passes through a point M_{i+1} at a certain value of the parameter u_{i+1}. Thus, the number of parameter u_{i+1} values corresponds to the number of points M_{i+1}. In [10], the distribution of the parameter was assumed to be uniform from 0 to 1, and in [11] it was assumed to be uneven and dependent on the coordinates of the initial points.

The use of Bezier curves is conditioned by the peculiarities of determining the coefficients on the basis of Newton's binomial for the current parameter u and its addition to 1, based on which the condition is true $\left(\bar{u}+u\right)^{n}=1$. The fulfillment of this condition ensures that the Bezier curve belongs to n-dimensional space, which allows us to generalize the interpolation curves to multidimensional space due to the invariant properties of the point equations with respect to parallel projection. And due to the invariant properties of point equations with respect to parallel projection, it allowed us to determine interpolation curves in a space of any dimension. At the same time, further research has shown that in this way it is possible to model interpolation curves, using as a prototype any continuous curves parameterized in point calculus.

As a result, a special program was developed in the Maple [20] computer algebra system to determine point equations of interpolation curves. The program is implemented in the Maple internal programming language. The following listing of the program is shown using the example of determining the point equations of an interpolation curve based on a Bezier curve of the n-order in accordance with the mathematical apparatus given above.

restart;
n:= 5; *e*:= 0;
for *i* **from** 0 **to** *n* **do**

$$eq[i] := A[i+1]\cdot\frac{n!}{i!\cdot(n-i)!}\cdot(1-u)^{n-i}\cdot u^{i}:$$

$$e := e + eq[i]:$$

od:
S:= {}: *SA*:= {}: *SM* := {}:
for *i* **from** 0 **to** *n* **do**

$$eq1[i] := subs\left(\left\{u = \frac{i}{n}\right\}, e\right) = M\left[i+1\right]:$$

$$S := S \bigcup eq \quad i$$

$$SA := SA \bigcup A \ i$$

$$SM := SM \bigcup M \ i \qquad :$$

od:
R:= *solve*(*S, SA*); *assign*(*R*);
collect(*e, SM*);

In this example, the input data is exclusively the curve order n, which defines the initial Bezier curve equation. In the first cycle, all necessary equations of the system *eq*[*i*] are formed on the basis of the initial Bezier curve equation. The second cycle performs substitution of parameter values on the interval from 0 to 1, as a result of which the system of linear algebraic equations is compiled and solved using the *solve* operator. The *collect* operator for convenience of further use performs sorting of the obtained equation by the interpolation curve represented in point form, where points are coordinate vectors.

By Similarly, by specifying the equations of other initial continuous curves *eq*[*i*] parameterized in pointwise calculus, one can obtain the pointwise equations of interpolation curves based on them.

3. Modeling of the bulbous dome surface

As one example of the use of interpolation curves with predetermined geometric properties, consider the process of modeling the surface of the onion dome. According to the geometrical conditions of the problem (Fig. 1), it is necessary that the forming line of the surface passes through the points M_1, M_2, M_3 and has a tangent M_1C_2 at the point M_1. The point M_1 is fixed and the points M_2 and M_3 are current, which form closed lines. Thus, the image is defined by two flat closed sections and the point at which the curve image touches the axis of the rotation surface. To prevent self-intersections of the dome surface, it is necessary to fulfill the condition of the tangency of the curve.

Proceeding from the fact that it is necessary to provide 4 geometric conditions (incidence of 3 points and 1 tangency), as an initial one we use the arc of the Bezier curve of the 3rd order, the flexibility of which is enough to provide the inflection of the forming line of the modeled surface [21]:

$$M = C_1\bar{u}^3 + 3C_2\bar{u}^2u + 3C_3\bar{u}u^2 + C_4u^3. \quad (2)$$

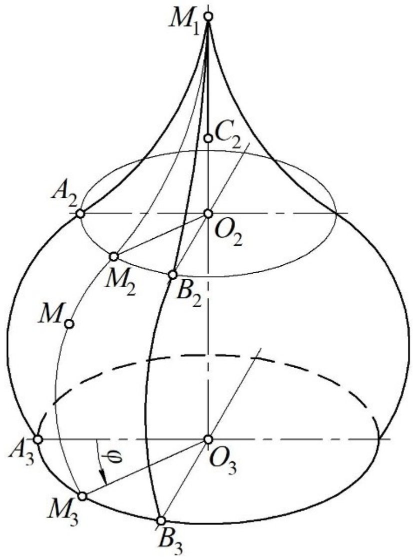


Fig. 1. Geometric scheme for modeling the surface of an onion dome

For each of the three points of the forming line M_1, M_2 and M_3 we take the following values of the current parameter u equal to $0, 0.5$ and 1. In this case, it is assumed that the section containing the line M_2 is in the middle. If it is located higher or lower, it is necessary to enter the corresponding value instead of the parameter $u = 0.5$ value. After substituting the parameter values into equation (2) and solutions to SLAEs, we obtain:

$$M = \bar{u}(1-2u)M_1 + 8u^2\bar{u}M_2 + u^2(2u-1)M_3 + 3\bar{u}u(1-2u)C_2. \quad (3)$$

The points in equation (3) are understood to be coordinate vectors that provide hidden parallelism as a result of the construction and visualization of the surface. When using parameterization, the corresponding coordinate vectors can be changed with additional parameters, taking into account geometric conditions. In the coordinate form, equation (3) will take the following form:

$$\begin{cases} x_M = \bar{u}(1-2u)x_{M_1} + 8u^2\bar{u}x_{M_2} + u^2(2u-1)x_{M_3} + 3\bar{u}u(1-2u)x_{C_2} \\ y_M = \bar{u}(1-2u)y_{M_1} + 8u^2\bar{u}y_{M_2} + u^2(2u-1)y_{M_3} + 3\bar{u}u(1-2u)y_{C_2} \\ z_M = \bar{u}(1-2u)z_{M_1} + 8u^2\bar{u}z_{M_2} + u^2(2u-1)z_{M_3} + 3\bar{u}u(1-2u)z_{C_2} \end{cases}.$$

To determine the lines M_2 and M_3, we will use the point equation of an ellipse with two conjugate axes [20] This is one of the simplest parameterizations of a circle in the case when the conjugate axes O_jA_j and O_jB_j are equal.

where $\varphi \in \left[0;2\pi\right]$ – the current angular parameter (Fig. 3).

In result is we obtain a computational algorithm for modeling the surface of the bulbous dome. It provides for the definition equations of moving points M_2 and M_3, followed by the construction of the forming surface of the bulbous dome. This surface is described by a continuous curve of order 3, incident to 3 points and having a tangent at the beginning of the arc. To visualize the resulting bulbous dome model, we will use the Maple computer algebra system (Fig. 2a). Also, as an example, we will model a bulbous dome, in which the line M_2 will not be a circle, but a sine wave with an axis in the form of a circle (Fig. 2b). Its algorithm for constructing which and the geometric scheme are described in [11]. At the same time, the equation of the generator remains unchanged.

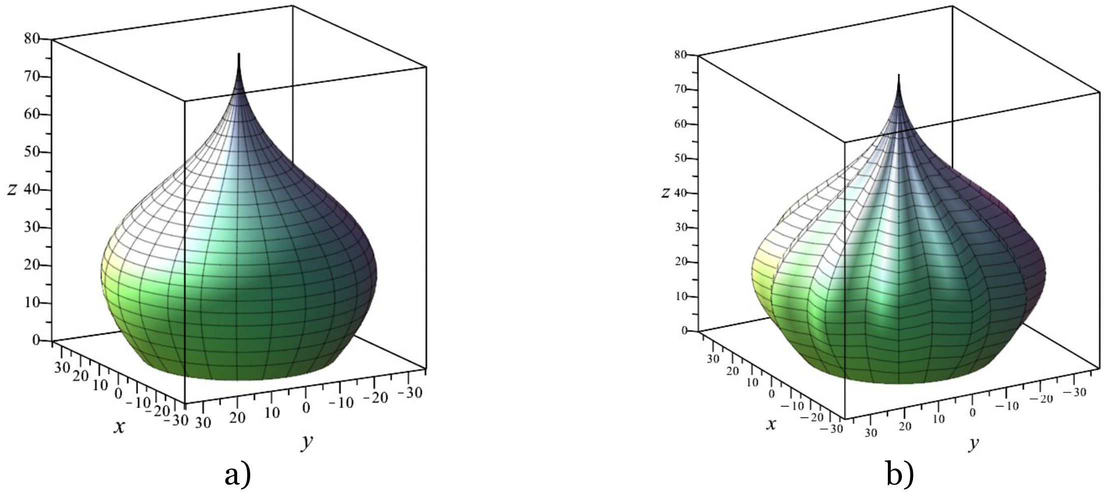


Fig. 2. Visualization of the bulbous dome surface: a) with a circle in the plane $O_2A_2B_2$; b) with a sine wave in the plane $O_2A_2B_2$

The proposed method can also be used to simulate the composite surfaces of the dome (Fig. 4). In this case, each of the lobes of the surface is determined by the kinematic operation of the movement of the forming curve of the 3rd order, which passes through 3 points and has a vertical tangent along two circles that are located in planes $O_2A_2B_2$ and $O_3A_3B_3$. The desired number of circles must be set in advance, and their radius is calculated depending on the lengths of the arcs of the guiding circles $A_2M_2B_2$ and $A_3M_3B_3$ in such a way that the surface of the bulbous dome is closed

An example of an element of such a composite surface is shown in Fig. 3. The algorithm for constructing this element does not fundamentally differ from the geometric scheme shown in Fig. 1 and its analytical description given above. Only the coordinates of the points differ M_1, O_2 and O_3. They are no longer incident to a single vertical line.

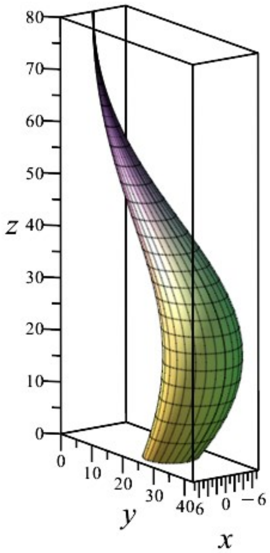
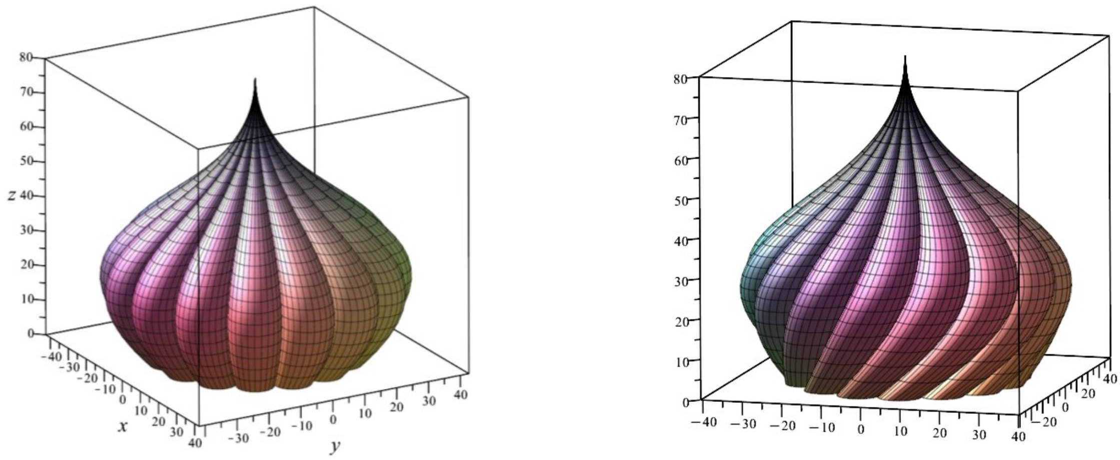


Fig. 3. The petal element of the composite surface of the bulbous dome

Next, to obtain a composite surface, it is necessary to fill the circular array with the resulting petal elements (Fig. 3). To do this, an external cycle is formed. The results of this operation are shown in Figures 4a and 4b.

It should be noted that in all cases, a continuous interpolation curve of the 3rd order was used as a generative one. It has a tangent, As the minimum possible algebraic curve to provide the necessary shape of the surface with a minimum of geometric conditions and the simplest mathematical apparatus, which is based on the use of coordinate vectors.



a) b)

Fig. 4. Visualization of modifications to the shape of the bulbous dome surface: a) with a vertical petal; b) with a twisted petal.

4. Surface modeling in the form of a bowl by sections with a sine wave

As another example of modeling a surface using interpolation curves, consider a bowl surface model. To compare the results, we will build a model of the form of a bowl surface in 2 ways: using continuous interpolation curves and using splines. In our example, all horizontal sections of the surface consist of circles, except for the plane $O_2A_2B_2$, which is defined by a sinusoid with a circular axis (Fig.5).

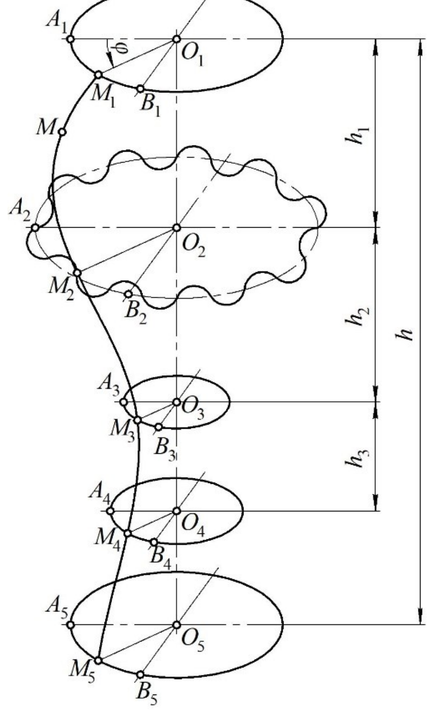


Fig. 5. Geometric scheme of the bowl-shaped surface

Equations of points M_1, M_3, M_4, M_5 differ only in the indices and have the following form:

$$M_j = (A_j - O_j)\cos\varphi + (B_j - O_j)\sin\varphi + O_j \tag{4}$$

The equation of the point M_2 – the sinusoid with a circular axis described in the work [11]:

$$\begin{aligned} N_2 &= (A_2 - O_2)\frac{\sin(\gamma-\varphi)\sqrt{1+m^2+2m\cos(m\varphi)}}{m\sin(\gamma)} + \\ &+ (B_2 - O_2)\frac{|A_2O_2|\sin(\varphi)\sqrt{1+m^2+2m\cos(m\varphi)}}{|B_2O_2|\,m\sin(\gamma)} + O_2, \end{aligned} \tag{5}$$

where m - the number of "waves" of the sine wave, and angle γ in this example is equal to $\frac{\pi}{2}$.

After determining the equations of the guide lines, in order to build a model on top of the news, it is necessary to substitute equations (4) and (5) into the equation of the generating interpolation curve, passing through 5 interpolation nodes M_i:

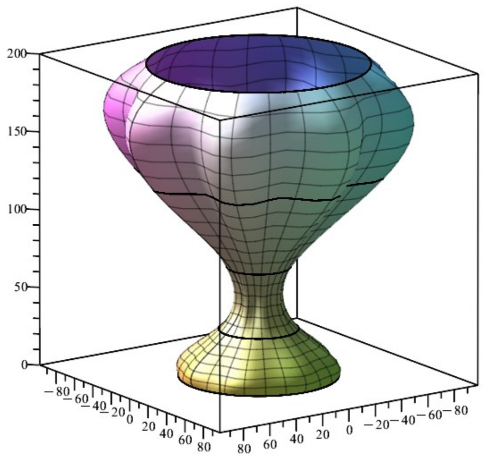
$$M = \left(\bar{u}^4 - 13\frac{\bar{u}^3u}{3} + 13\frac{\bar{u}^2u^2}{3} - \bar{u}u^3\right)M_1 + \left(16\bar{u}^3u - 64\frac{\bar{u}^2u^2}{3} + 16\frac{\bar{u}u^3}{3} - \bar{u}u^3\right)M_2 +$$

$$+\left(-12\bar{u}^3u + 40\bar{u}^2u^2 - 12\bar{u}u^3\right)M_3 + \left(16\frac{\bar{u}^3u}{3} - 64\frac{\bar{u}^2u^2}{3} + 16\bar{u}u^3\right)M_4 + \left(-\bar{u}^3u + 13\frac{\bar{u}^2u^2}{3} - 13\frac{\bar{u}u^3}{3} + u^4\right)M_5,$$

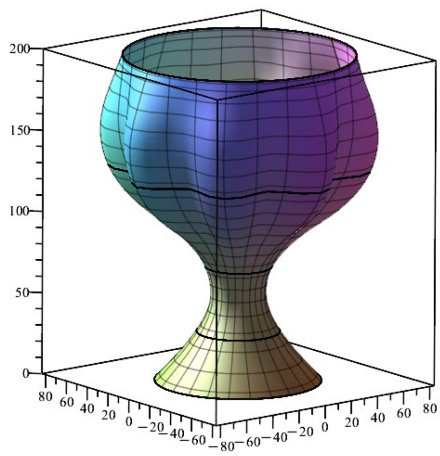
where u – a parameter that varies from 0 to 1.

It should be noted that this equation of the continuous interpolation curve of the 4th order passing through 5 interpolation nodes is obtained using a special program implemented in the Maple computer algebra system, the listing of which is given above.

The result of modeling the desired surface is shown in Fig. 6a. And Fig. 6b shows the result of modeling the surface of the bowl based on the same geometric scheme (Fig. 5), in which a spline of the 2nd order of smoothness was used as a generatrix instead of an interpolation curve, which forms a composite spline surface. As can be seen from Fig. 6, the model based on interpolation curves inherits traces of the "waves" of the sine wave to the base of the surface. And the spline surface in all 4 sections has retained the shape of a circle, and traces of "waves" are preserved only in the area between the planes $O_1A_1B_1$ and $O_3A_3B_3$.



a)



b)

Fig. 6. Visualization of modifications to the shape of the vase surface: a) with a continuous interpolation curve forming; b) with a spline forming

Thus, the results of modeling the two variants of the vase surface are quite different from each other. The choice between using interpolation curves and splines can be made based on the geometric properties that the simulated surface should have. But it is also possible to combine it with spline surfaces, given the possibility of controlling the geometric properties of continuous interpolation surfaces. In addition, continuous interpolation arcs of the outline can be used to reduce the "piecemealness" of composite curved lines.

5. Conclusions

As a result of the conducted research, the following conclusions can be drawn:

1. The process of surface modeling presented in the paper by the method of the movable simplex of point calculus, taking into account the possibility of generalization to a multidimensional space, is an expanded analogue of the cross-section operation (lofting operation), which is widely used in automated design and visualization systems.
2. Both continuous and composite interpolation curves can be used as forming lines of such surfaces, as shown in the examples. The choice of the interpolation method depends on the conditions of a specific practical task. At the same time, in our opinion, continuous interpolation curves, as well as piecewise ones, have the versatility necessary for computer-aided design systems and could significantly complement their tools along with splines.
3. The program for determining the point equations of continuous interpolation curves using coordinate vectors is shown in the example of using Bezier curves as a prototype. But after minor modifications, as shown in the examples, these curves can be adapted both to the initial data and to the necessary geometric conditions. This significantly expands the possibilities of their use in CAD.
4. The proposed approach to modeling surfaces of complex shape provides for generalization towards parameterization of geometric bodies as three-dimensional objects belonging to three-dimensional space [22].
5. The implementation of any complex shaping tools in CAD requires a fairly large amount of computing resources. The features and benefits of point calculus, taking into account hidden parallelism, can provide significant performance gains for projects containing large volumes of geometric elements.

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